Management Science

Short- or Long-Duration Coupons: The Effect of the Expiration Date on the Profitability of Coupon Promotions

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United States firms collectively spend over $6.5 billion annually on coupon promotions and are becoming increasingly concerned with their profitability. FSI (free-standing-insert) data show that coupon duration varies across brands. In this paper, we show how coupon duration can affect coupon profitability. We also provide answers for some empirical observations on coupon duration. We explain, for example, why (i) coupon duration will vary across firms, such that large market share firms will give short-duration coupons and small market share firms will give long-duration coupons; (ii) longer coupon duration for one brand will increase redemption for coupons of that brand and of a competing brand; (iii) coupon duration will affect coupon profitability.
(*Promotion*; *Competitive Strategy*; *Buyer Behavior*)

1. Introduction

U.S. firms collectively spend over $6.5 billion annually on coupon promotions (Leclerc and Little 1997), and are thus justifiably concerned with their putative profitability (*Wall Street Journal* 1996, 1997). Procter and Gamble (P&G), for example, recently conducted three test markets in upstate New York to investigate a question as basic as whether coupons should be abandoned entirely. While P&G concluded that such an abandonment would not, at least in the near future, be a sound course of action, this nonetheless leaves open the question of how couponing profitability can be enhanced or potentially optimized.[1]

This paper focuses on a pivotal aspect of the couponing phenomenon: that of duration. At present, over 99% of coupons impose a fixed expiration date.[2] Yet, for a mechanism so ubiquitous, very little is known about consumer reaction to varying coupon duration, let alone how it might be most reasonably set. For example, the recent NCH NuWorld survey indicates that over 70% of consumers feel that coupons expire too quickly.[3] Inman and McAlister (1994) provide evidence that expiration dates serve to influence redemption behavior, in that redemption rate increases markedly just prior to expiration. Promotion "experts" routinely advise their corporate clients to employ short-duration coupons for faster sales and long-duration coupons for higher total sales (*Food & Beverage Marketing* 1991). Furthermore, FSI (free-standing-insert) data indicate that coupon duration indeed varies sub-

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[1] P&G announced the initiative in April, 1996. A year later (April 13, 1997), *Wall Street Journal* reported that P&G had declared the test market a success, but would reinstate all coupons.

[2] Throughout, coupon duration is taken as the time from drop to expiration.

[3] This and related information has been provided by NCH NuWorld Marketing Limited, and appears in the *1998 World-Wide Coupon Distribution and Redemption Trends*.

Table 1   Summary Statistics of FSI Data*

Variable/Category	Cereal	Spaghetti Sauce	Hot Dogs
Mean Face Value ($)	0.69	0.47	0.49
	(0.26)	(0.53)	(0.59)
Mean Price ($)	2.9	0.87	1.72
	(0.57)	(0.16)	(0.29)
Mean Duration (Days)	68.6	63.7	106.8
	(46)	(30.4)	(61.9)
Sample Size	344	522	209

* Standard deviation is reported in parentheses.

stantially across brands (Table 1). There is presently little in the way of a theoretical apparatus for examining, comprehending, or predicting such widely observed effects.

In this paper, we formulate a model to clarify the role of duration on coupon profitability. Among the major findings to result from the model is that it is more profitable for large-share firms to offer short-duration coupons, while the reverse is true for small-share ones.

While prior research has focused on coupon face value (Bawa and Shoemaker 1989, Leone and Srinivasan 1996), coupon distribution vehicle (Neslin and Shoemaker 1983, Raju et al. 1994, Dhar et al. 1996), coupon frequency (Gonul and Srinivasan 1996, Neslin 1990), and coupon redemption rate (Reibstein and Traver 1982, Ward and Davis 1978), it has not focused on expiration date. A study of expiration date effects can provide answers to many empirical observations. For example, How does the expiration date affect individual coupon redemption? If the only effect of a shorter expiration date is to accelerate redemption, then this cannot account for the large variation in coupon duration. Thus, what other effect, besides speedy redemption, does shorter coupon duration have on individual coupon redemption? Why does coupon duration vary across brands? Most importantly, can coupon duration affect coupon profitability, and in what manner?

In this paper, we explain, for example, why (i) coupon duration will vary across firms, such that large market share firms will give short-duration coupons and small market share firms will give long-duration

coupons; (ii) longer coupon duration for one brand will increase redemption for coupons of that brand and of a competing brand; (iii) coupon duration will affect coupon profitability.

From a theoretical perspective, studying coupon duration is also important. Coupon duration differs from other familiar aspects of price promotions. It lengthens the time of a price discount, but unlike a longer price deal in a store, the discount can only be used once. Also, unlike an increase in price discount or in coupon face value, there is no increase in the magnitude of the deal itself when coupon duration is increased. A theoretical investigation reveals the role of coupon duration in promotions, and sheds some light on the more general question of how time pressure can be used effectively as a competitive marketing strategy.

The rest of the paper is organized as follows: We first examine FSI coupons (free-standing-insert) to establish the relationship between coupon duration and other factors (§2). We then build a stylized model to describe how a rational consumer would redeem coupons (§2). Based on this model, we analyze how individuals would respond to changes in coupon duration of two competing firms (§3). Individual behavior is then aggregated to the market level and explanations are provided for the empirical observations in §2 (§4). We conclude with a discussion of the model's limitations.

2.   An Examination of FSI Coupons

We conducted an analysis of FSI coupons for three product categories: breakfast cereal, spaghetti sauce, and hot dogs.[4] Data for breakfast cereal was obtained for the St. Louis market for 1993 and 1994. Spaghetti sauce data was obtained for Chicago, Los Angeles, and New York, and hot dog data for Atlanta, Cincinnati, Detroit, Tampa, and Los Angeles, both for 1989 and 1990. The analysis was done at the brand-variety level (e.g., Prego Extra Chunky Spaghetti Sauce). There were seven brand varieties for hot dogs, 127 for cereal, and 11 for spaghetti sauce. There was wide variation in coupon duration for different brand vari-

[4] We thank A. C. Nielsen for providing some of this data.

Table 2   FSI Data Analysis: Effect of Market Share on Coupon Duration*

Cereal		Spaghetti Sauce		Hot Dogs	
Variable	Coefficient	Variable	Coefficient	Variable	Coefficient
Market Share	−3.21	Market Share	−2.57	Market Share	−3.33
	(−2.28)		(−3.04)		(−3.53)
(Face Value)/Price	8.47	(Face Value)/Price	−9.87	(Face Value)/Price	−40.83
	(0.38)		(−6.77)		(−3.68)
Coupon Frequency	−0.89	Coupon Frequency	−0.07	Coupon Frequency	0.87
	(−0.49)		(−0.63)		(0.82)
Hot	27.29	CHI	−1.55	ATL	−14.95
	(1.28)		(−0.45)		(−0.71)
Sweet	10.70	NY	4.33	CIN	−25.00
	(1.99)		(1.43)		(−1.18)
QO	−27.33	BF	−1.44	DET	−12.43
	(−0.79)		(−0.23)		(−0.59)
RP	−38.51	CP	1.01	TAM	−17.37
	(−1.08)		(0.15)		(−0.82)
KL	−34.04	CS	−39.36	BF	−60.95
	(−1.01)		(−8.71)		(−2.17)
GF	−48.54	FR	32.35	OM	−14.05
	(−1.42)		(5.74)		(−0.46)
GM	−27.28	UB	0.96	BM	−75.42
	(−0.81)		(0.20)		(−2.06)
RJR	−37.75			LB	89.83
	(−1.08)				(2.55)
Adjusted R^2	0.024	Adjusted R^2	0.57	Adjusted R^2	0.31
					209
Sample Size	344	Sample Size	522	Sample Size	

* *t*-stat is reported in parentheses. Coupon duration is the dependent variable. Hot and Sweet are category dummies based on the classification in IRI *Marketing Fact Book.* CHI, NY, ATL, CIN, DET, and TAM are market dummies, with LOS as the default. The rest of the variables are manufacturer dummies for Quaker Oats (QO), Ralston Purina (RP), Kellogg (KL), General Food (GF), General Mill (GM), RJR Nabisco (RJR), Beatrice Foods (BF), Chesebrough-Ponds (CP), Campbell Soup (CS), Francesco Rinaldi Food (FR), United Biscuits (UB), Oscar Mayer (OM), Bil-Mar Foods (BM), and Lykes Bros (LB). Malt-O-Meal, Borden, and Farmer John are used as the defaults for cereals, spaghetti, and hot dogs, respectively.

eties for these three product categories as shown in Table 1.

We regressed coupon duration of each coupon against market share of that brand variety, face value of coupon adjusted for mean price of the brand variety, and coupon frequency (number of coupons dropped) for that brand variety.[5] Since breakfast cereal is divided into many subcategories we also included subcategory dummies for cereal—there were

three subcategories per *The Marketing Factbook*: regular cold cereal, sweetened cold cereal, and hot cereal. Since spaghetti sauce and hot dog data were for different geographic markets, we used dummies for the different markets. Furthermore, to control for manufacturer specific effects, we also included manufacturer dummies for each product category. Regression results are reported in Table 2.[6]

Table 2 clearly shows that *in all three product categories, market share has a significant negative effect on coupon*

[5] Market share and mean price for each brand in the regression analysis are obtained from *The Marketing Factbook* for the same year as the coupon was delivered.

[6] We thank an anonymous reviewer and the area editor for suggesting this analysis.

duration, *i.e.*, *the higher the market share, the lower the coupon duration*. Face value (adjusted for price) significantly and negatively affects coupon duration for hot dogs and spaghetti sauce, but the effect is not significant for cereals. Coupon frequency has no significant effect on coupon duration for all three product categories. There are also some significant city, subcategory, and manufacturer-specific effects.

3. A Model of Individual Coupon Redemption

In spite of extensive prior literature on couponing, little research has studied how an individual consumer redeems coupons. On the other hand, Neslin (1990) has pointed to the need to study individual coupon redemption behavior since it "has the potential to uncover more fundamental parameters that are driving the purchase/redemption decision". In this section, we fulfill that need and propose a stylized model of how individual consumers redeem coupons.

3.1. Assumptions
We assume that the market consists of two competing brands, A and B, both of which use coupon promotions.

Coupon Frequency. Similar to Raju et al. (1994) and Dhar et al. (1996), the probability that the consumer receives a coupon for brand i on a particular purchase occasion is assumed to follow a Bernoulli process with parameter a_i, such that $0 < a_i < 1$. This implies that the time (number of periods) until the next coupon is received is a memoryless geometric distribution, and there is a nonzero probability of receiving a coupon for brand i in each period. This assumption rules out strategies where the probability of coupon availability for brand i is lower immediately after receiving a coupon for brand i. In Technical Appendix D, we discuss why this assumption is not restrictive for our implications.[7]

Coupon Duration. To examine the effect of coupon expiration date on coupon redemption, we con-

trast *short-* (one period) and *long-* (two periods) coupon duration. A duration longer than two periods, if contrasted with the base duration of one period, would make our results stronger.[8] We assume for simplicity that all coupons for Firm i have the same expiration date, either short or long. We also assume that there is no interaction between the probability of getting a coupon and coupon duration. This can be interpreted as consumers getting a one-shot random chance of getting the coupon in the Sunday paper. Customers do not get "old" coupons. This seems like a reasonable assumption except for transactions at coupon-exchange clubs.

Coupon Processing Cost. We assume that processing costs for all coupons are zero. In reality, processing costs may vary across the population so that some consumers choose to be coupon users while others do not. Since coupon duration only influences coupon users and the change in coupon duration does not change the costs involved in clipping, storing, and retrieving coupons, this assumption does not affect our results.

Purchase Frequency. Consumers purchase one unit of a brand in each period.[9] This assumption rules out the possibility that consumers redeem coupons to stockpile the product. While relaxing this assumption will not qualitatively change our conclusions as we will discuss in conclusion, prior research indicates that coupons lead to low or negligible stockpiling. Krishna and Shoemaker (1992) examined coupons of three different face values for a brand from a field experiment. They found no significant difference in package size purchased, number of units purchased or total quantity purchased with and without coupons. Neslin et al. (1985), examining incremental sales from cou-

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[7] Technical Appendices D, E, and F are available from the authors upon request.

[8] Note that while coupon arrival is a zero-order process, the consumer's inventory state is first-order since coupon duration can be two periods long.

[9] Explicitly allowing for heterogeneity in interpurchase timing should also not change our results—there will of course be heterogeneity in consumer interpurchase times so that the same coupon duration may imply a different number of purchase periods for different consumers. However, given a population of consumers, the number of periods for coupon duration may apply in an average sense.

Figure 1   Model of Individual Coupon Redemption

Use B's Coupons Only   Use Coupons for Both Brands   Use A's Coupons Only

$-f$ f d_j
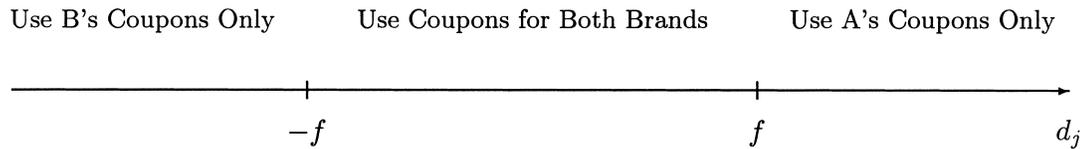
pons, found coupon purchases to be 17% higher for bathroom tissue and 7.9% higher for coffee (p. 158). However, many of the coupons stipulated a minimum purchase quantity. They stress that "this by itself would induce at least some acceleration of quantity." The coupon frequency and purchase frequency assumptions also imply that consumers receive less than one coupon for their preferred brand in each interpurchase cycle. While we believe this to be true for most product classes, it need not be true for some product classes.[10]

Brand Preference. Consumer j is assumed to prefer Brand A to Brand B by d_j such that if she prefers Brand A, $d_j > 0$, if she prefers Brand B, $d_j < 0$, and if she is indifferent between the two brands, $d_j = 0$. The magnitude of d_j is defined as the difference in price between the preferred and less preferred brands necessary to induce the consumer to switch from the preferred to the less preferred brand.[11]

Coupon Face Value. For purposes of tractability, we assume that all coupons for Firm i have a face value f_i (i.e., $f_A = f_B = f$). Different face values can easily be incorporated in the model by adjusting brand preference.

We do not consider the effect of price deals and other in-store promotions on coupon redemption since our focus is on isolating the effects of coupon duration.

3.2. Modeling Individual Coupon Redemption Decision
Given our simplification of zero processing cost, coupons for the consumer's preferred brand are always redeemable. However, the consumer will not redeem

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[10] We thank an anonymous reviewer for pointing this out to us.

[11] The same notion is used by Pessemier (1959), Narasimhan (1988), Raju, Srinivasan, and Lal (1990), Krishna (1992).

a coupon for the less preferred brand if its face value is smaller than the consumer's switching cost ($|d_j|$). Brand preference thus segments coupon users into three types (see Figure 1): (i) those who only use coupons for Brand B ($d_j < -f$); (ii) those who use coupons for both brands ($-f \le d_j \le f$); and (iii) those who only use coupons for Brand A ($d_j > f$). Consumers who only redeem Brand A's coupons or only redeem Brand B's coupons are not affected by a change in expiration date. These consumers redeem a coupon for Brand A (or Brand B) whenever they get it, regardless of expiration date. However, consumers who redeem coupons for both brands are affected by a change in expiration date. We focus our analysis in the rest of the paper on this latter group of coupon users.

There are four scenarios in which a consumer who redeems coupons for both brands must make a redemption decision. We first state three simple decisions a rational consumer will make.

• A consumer who has only a solitary coupon on a purchase occasion will optimally redeem it without delay.

• A consumer who has multiple coupons of different brands with the same expiration date on a purchase occasion will redeem them in order of brand preference.

• A consumer who has an expiring coupon for the preferred brand and a nonexpiring coupon for the less preferred brand will redeem the former.

A more complicated redemption decision arises when a consumer must decide whether to redeem a coupon for the preferred brand that expires in the next period or a coupon for the less preferred brand that expires in the current period. The choice for the coupon user in this scenario is which of the overlapping coupons to redeem—whether to redeem the coupon for the preferred brand immediately (i.e.,

Table 3   Long-Term Purchase Probabilities for Different Coupon Redemption Policies*

Purchase Probabilities for Consumers who Redeem in "Order of Brand Preference"

Brand Preference	Buy A With Coupon	Buy B With Coupon	Buy Preferred Brand Without Coupon
	Environment: Short Duration Coupon for Both Brands		
Brand A	a_A	$a_B(1 - a_A)$	$1 - a_A - a_B + a_A a_B$
Brand B	$a_A(1 - a_B)$	a_B	$1 - a_A - a_B + a_A a_B$
	Environment: Long Duration Coupon for Brand A, Short Duration Coupon for Brand B		
Brand A	a_A	$a_B(1 - a_A)$	$1 - a_A - a_B + a_A a_B$
Brand B	$(1 - a_B)[p_A^0 + a_A(1 - p_A^0)]$	a_B	$(1 - a_B)[1 - p_A^0 - a_A(1 - p_A^0)]$

Purchase Probabilities for Consumers who Redeem in "Order of Expiration Date"

Brand Preference	Buy A With Coupon	Buy B With Coupon	Buy Preferred Brand Without Coupon
	Environment: Short Duration Coupon for Both Brands		
Brand A	a_A	$a_B(1 - a_A)$	$1 - a_A - a_B + a_A a_B$
Brand B	$a_A(1 - a_B)$	a_B	$1 - a_A - a_B + a_A a_B$
	Environment: Long Duration Coupon for Brand A, Short Duration Coupon for Brand B		
Brand A	a_A	$a_B(1 - p_A^0)$	$1 - a_A - a_B + a_B p_A^0$
Brand B	$(1 - a_B)[p_A^0 + a_A(1 - p_A^0)]$	a_B	$(1 - a_B)[1 - p_A^0 - a_A(1 - p_A^0)]$

* a_i is the probability for a coupon user to receive a coupon for brand i, and p_A^0 is the steady-state probability of holding an expiring coupon for Brand A by those who prefer the rival brand ($a_A a_B/(1 - a_A + a_A a_B)$).

redeeming *in order of brand preference*), or to redeem the expiring coupon for the less preferred brand and save the nonexpiring preferred brand coupon for a later use (i.e., redeem *in order of expiration*).

Both of these two rules may be optimal for a rational consumer (proof in Technical Appendix A), depending on various exogenous factors, such as the magnitude by which an individual consumer discounts future utility. (*To know precisely which consumer uses which redemption policy is not essential for our analysis. Our analysis merely requires that both types of consumers exist in the market.*)

4. Coupon Duration and Purchase Behavior: Comparative Statics

We now use the individual coupon redemption model to analytically derive the steady-state purchase probabilities when Firms A and B have either a short or a long expiration date. Remember that consumers who redeem *in order of brand preference* redeem a coupon for their preferred brand whenever they have one. Hence, they choose to redeem a nonexpiring coupon for the preferred brand over an expiring coupon for the less

preferred brand. Consumers who redeem *in order of coupon expiration*, however, redeem an expiring coupon for the less preferred brand over a nonexpiring coupon for the preferred brand.

By varying a brand's coupon duration from *short* to *long* while keeping the competing brand's duration constant at either *short* or *long*, we can see how a brand's coupon duration affects purchase decisions for consumers who follow the two different redemption rules. Table 3 shows how purchase probabilities change when Brand A increases coupon duration from short to long, while Brand B has a short-duration coupon. (In Technical Appendix B, we derive similar results for the case where Brand B's duration is long.) We summarize our analysis in the following propositions.

PROPOSITION 1. *For consumers who redeem in order of brand preference, a longer duration coupon for Brand A will attract some of Brand B's customers without losing any of Brand A's customers.*

This proposition follows from the difference between relevant cells in Table 3 (Rows 4 and 6). What

happens if the manager increases coupon duration for Brand A when Brand B's coupons have a short duration? Consumers who prefer Brand A will continue to redeem all the coupons they get for Brand A, and will redeem a coupon for Brand B when they have no coupon for Brand A. They will make noncoupon purchases for Brand A when they have no coupons for Brand A or Brand B.

For consumers who prefer Brand B, however, redemption of Brand A's coupons will increase. Coupon purchases for Brand A increase because a long coupon-duration for Brand A allows consumers who prefer B to use Brand A's coupons on a greater number of occasions when they do not have a coupon for Brand B. Since a greater number of coupons are redeemed by these consumers, the noncoupon purchases of their preferred brand (Brand B) decrease. Table 3 (Rows 4 and 6) shows that redemption of Brand A's coupons from those who prefer Brand B increase from $(1 - a_B)a_A$ to $(1 - a_B)(p_A^0 + a_A(1 - p_A^0))$. Here, p_A^0 is the steady-state probability of holding an expiring coupon for Brand A by those who prefer Brand B.[12] Noncoupon purchases for Brand B decrease by the same amount as the coupon purchases for Brand A increase.

From a manager's perspective, therefore, if consumers redeem coupons in order of brand preference, increasing the duration for the brand's coupons will generate more incremental coupon sales without losing noncoupon sales. Thus, there is no trade-off for a manager in increasing coupon duration when consumers redeem in order of brand preference. The rival brand's manager, on the other hand, needs to watch out for the increase in the coupon duration of the competing brand, since it implies a decrease in noncoupon sales of his own brand.

PROPOSITION 2. *Among coupon users who redeem in order of expiration date, a longer duration coupon for Brand*

A will attract some of Brand B's customers while losing some of Brand A's customers.*

Proposition 2 also follows from the difference between relevant cells in Table 3 (Rows 9 and 11). As one would expect, when Brand A offers a longer duration, consumers who prefer Brand B have a greater chance to redeem coupons for Brand A and, hence, the overall redemption for Brand A's coupons increases. Surprisingly, however, Brand B's overall redemption rate also increases—if Brand A's coupons have a longer duration, consumers who prefer Brand A will be subject to less time pressure to redeem Brand A's coupons. Hence, they will have a greater chance to interject Brand B's coupons in their purchases. Therefore, redemption of Brand B's coupons from those who prefer Brand A will increase. Since coupon purchases of both brands increase, noncoupon purchases of both brands should decrease.

Thus, from a manager's perspective, if consumers redeem in order of expiration date, a long coupon duration for either his own brand or a rival brand should generate more incremental coupon sales. However, such incremental sales come only at the cost of losing noncoupon sales. Therefore, when consumers redeem in order of expiration date, increasing coupon duration involves a trade-off for the manager in coupon versus noncoupon sales. Note that the opportunity cost of lengthening coupon duration comes about due to the probability that "loyal" customers hold two overlapping, competing coupons.

This individual consumer model provides the foundation for our market-level analysis of sales and coupon profitability.

5. Market-Level Analysis

5.1. Effect of Coupon Duration on Aggregate Sales

Putting Propositions 1 and 2 together, we draw the following conclusion.

PROPOSITION 3. *Given both types of consumers in the market, as a brand prolongs its coupon duration,*
 (a) *the redemption rate for its coupons will increase,*
 (b) *coupon purchases of the rival brand will increase,*
 (c) *noncoupon purchases of both brands will decrease.*

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[12] If coupon users who prefer Brand B hold an expiring coupon for Brand A at time t, they must have had this coupon in the previous period $t - 1$ and not have used it. Then, they must have optimally chosen to use a different coupon at time $t - 1$, which implies that they had an expiring coupon for either Brand B or Brand A in that period. Thus, $p_A^0 = a_A[a_B + (1 - a_B)p_A^0]$, which yields $p_A^0 = a_A a_B / (1 - a_A + a_A a_B)$.

Managerial Trade-off for Coupon Duration Decisions. Proposition 3 uncovers some interesting trade-offs a manager needs to make in setting the coupon duration. The proposition shows that the desirability of long-duration versus short-duration coupons hinges on the trade-off between increasing coupon redemption to consumers who prefer the rival brand, and decreasing noncoupon purchases to consumers who prefer the brand.

Thus, competing firms in a market may face different incentives to use long-duration or short-duration coupons. A firm with a large share of consumers preferring its brand has relatively few new buyers to attract. Everything else being equal, greater coupon sales to new buyers through long-duration coupons are not likely to outweigh the loss of noncoupon sales to the rival brand. This may spur the firm to use short-duration coupons. On the other hand, a firm with a small share of consumers preferring its brand may choose to use long-duration coupons as the gain from coupon sales to new buyers is likely to exceed the loss of noncoupon sales resulting from long-duration coupons.

Trade-off for Coupon Duration Decision Versus Trade-off for Deal Magnitude. Clearly, the trade-off for increasing coupon duration is not the same as the trade-off for increasing the magnitude of a price deal or the face value of a coupon. When increasing deal magnitude or face value, the manager trades off the loss from an increased discount to consumers who would have purchased the product without further discounting, with the gain from attracting new buyers. In increasing coupon duration, however, the trade-off is between losing sales to consumers who prefer the brand, and increasing sales to consumers who prefer the rival brand. This loss in sales occurs because consumers who prefer the brand and were purchasing the brand at full price now purchase the rival brand with a coupon.

Trade-off for Coupon Duration Decision Versus Trade-off for Deal Frequency. Our results may appear similar in nature to models concerning frequency of price deals. Raju et al. (1990), for example, show that brands with larger loyalty will offer less frequent discounts than smaller share brands. The intuition for the results found by Raju et al. is, however, close to that for increasing deal magnitude given in the previous paragraph, and not similar to the intuition for our results on coupon duration—when increasing deal frequency, the manager trades off the loss from lower unit margins from consumers who would have purchased the product without further discounting, with the gain from attracting new buyers. A brand with higher loyalty has less to gain from offering discounts.

5.2. Aggregate Market Profits: Competitive Context
Until now we have done a comparative statics analysis, where we have varied the coupon duration of one firm's coupons at a time. We now do a competitive analysis where we permit both firms to choose their coupon duration simultaneously. We allow the competing firms to repeatedly choose their coupon duration, either *short* or *long*, in an infinitely repeated game with the payoffs at each stage being based on the steady-state purchase probabilities derived in the preceding section. We limit our attention to the stage equilibrium of the infinitely repeated game. It is well known in game theory that each player "playing its Nash strategy of the stage game from now on" constitutes a subgame-perfect equilibrium for the infinitely repeated game (Fudenberg and Tirole 1991, p. 149).

To determine the profit implications of changing coupon duration for a brand, we aggregate across consumers who "prefer Brand A/prefer Brand B," and who "redeem in brand-preference-order/redeem in expiration-date-order." Let α denote the fraction of consumers who redeem coupons by brand preference, and $(1 - \alpha)$ the fraction of coupon users who redeem coupons by expiration date. Among each type of consumer, let θ fraction of them prefer Brand A and $(1 - \theta)$ prefer Brand B. To focus on the impact of coupon duration and to avoid unnecessary technical complications, we let the coupon arrival rates be equal so that we have $a_A = a_B = a$. We also let $m_A = m_B = m$ be a firm's sales margin net of coupon face value.

Aggregating across the consumer segments, we can derive the market-level profits for a firm in the different coupon duration environments (e.g., short-duration coupons for both brands), and make the following conclusion (see Technical Appendix C).

Figure 2   Profitability of Short-Duration versus Long-Duration Coupons
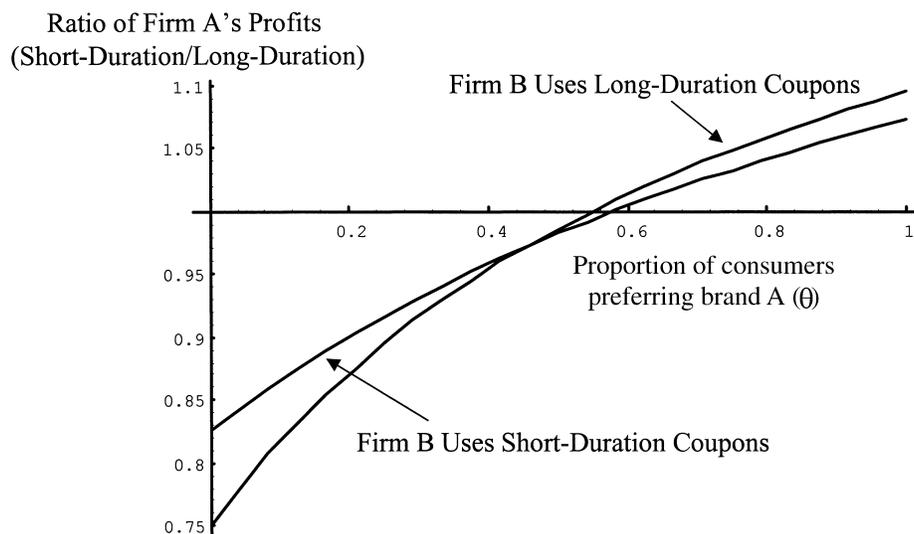
The firm's margin net of coupon face value, m=1; coupon face value for both brands, f=0.15; coupon arrival probability for both brands, a=0.5; fraction of coupon users who redeem by brand preference, α=0.3.

PROPOSITION 4. *As long as some coupon users redeem their coupons on the basis of brand preference and others do so on the basis of expiration date, for maximum profit,*

(a) *a brand with a sufficiently large number of consumers preferring it will set a short-coupon duration, whereas*

(b) *a brand with a sufficiently small following will set a long duration, and*

(c) *a brand is more likely to set a short duration as the proportion of consumers redeeming by expiration date increases (i.e., α is smaller).*

What drives Propositions 4(a) and 4(b) is the trade-off for duration decisions we have identified previously. When a firm has a sufficiently large share of coupon users preferring its brand, long-duration coupons draw relatively few new buyers, but cause substantial loss of noncoupon sales to the rival firm. Thus, short-duration coupons are preferable for the large-share firm. As illustrated in Figure 2, Firm A's profits from short-duration coupons exceed those from long-duration coupons when the share of cou-

pon users preferring its brand (θ) is sufficiently large, regardless of whether the rival firm uses short-duration coupons or long-duration coupons.

Proposition 4(c) states that the higher the proportion of consumers who redeem by expiration date, the more likely it is that short-duration coupons will be optimal for a firm. From Proposition 1, we know that if consumers redeem in order of brand preference, then there is no trade-off for the manager in increasing coupon duration—the firm is always better off with a long-duration coupon. However, from Proposition 2, we know that when consumers redeem in order of expiration date, the manager faces a trade-off in extending coupon duration. Hence, the greater the proportion of consumers redeeming in brand preference order, the better it is for the manager to have longer duration. Conversely, the higher the proportion of consumers who redeem by expiration date, the more likely it is that short-duration coupons will be optimal for a firm. (A formal proof is provided in Technical Appendix C.)

6. Explanations for Empirical Observations

Our theoretical model provides an explanation for the questions we started with:

Can Expiration Dates be Used to Increase Coupon Profitability? The analysis in the preceding section (especially as shown in Figure 2) clearly demonstrates that it can. In Figure 2, as the proportion of consumers preferring Brand A increases, it becomes more profitable for Firm A to issue short- (as opposed to long-) duration coupons. One can also see that this result is independent of Firm B's couponing strategy. Thus, our analysis shows that it is important for the manager to heed coupon duration in designing couponing promotions, and that it is beneficial to experiment with coupon duration.

Why Does Coupon Duration Vary Across Brands? We show that in a competitive context, coupon duration will vary across brands, such that larger market share firms give shorter coupon duration than small market share firms. This is consistent with the empirical data we presented.

Why has Coupon Duration Been Decreasing Over Time? The most recent NCH data show that, in 1997, nearly three quarters of all grocery coupons have a redemption deadline of only one to three months, and only 0.6% have no expiration date. The average duration for all coupons has steadily decreased from 10 months in 1980 to 3.1 months in 1993 (NCH 1994), holding mostly steady thereafter. As a byproduct, our model also suggests two plausible answers to this puzzling trend.

First, since large market share firms have a greater incentive to issue short-duration coupons, the average coupon duration will decrease over time if these firms issue proportionally more coupons over time. While we have no direct data to support this hypothesis, the fact that there has been considerable consolidation happening in the grocery trade (see Messinger and Narasimhan 1995) is consistent with the hypothesis.

Second, if more consumers redeem coupons by expiration-date order, by Proposition 4, firms will have more incentive to distribute shorter duration coupons. Consumers will be more likely to redeem by

expiration date if they are more keen to use all their coupons. Many factors may be conducive to consumers being more keen to use all their coupons. For instance, one factor may be attributable to the fact that the real income for the coupon-prone consumer segment (Bawa and Shoemaker 1987) decreased or stagnated between 1984 and 1994 (Kacapyr 1996), picking up appreciably only since 1995.

An alternative explanation for decreasing coupon duration may be that companies want to reduce their financial liability. While this a good reason for having expiration dates (versus not having expiration dates on coupons), it does not explain why coupon duration should decrease over time.

7. Conclusions and Limitations

In this paper, we set out to highlight the importance of coupon duration for coupon profitability. We first provided some empirical facts regarding coupon duration. Then, we proposed an individual-level model for coupon redemption. We used the model to examine how duration of coupons for competing brands should affect individual coupon redemption and purchase probabilities. By aggregating individual behavior to the market, we examined the incentives a firm faces in setting its coupon duration. The model is able to provide economic explanations for the empirical observations about coupons, and suggests some ways for firms to increase coupon profitability.

The key managerial insight from our analysis is that firms can use expiration dates as a strategic variable to change the mix of buyers. Increased coupon duration for a brand will increase coupon redemption for the brand, and also for the competing brand, at the expense of regular purchases for both brands. A brand with a high share of preference will find it more profitable to use short-duration coupons. However, a brand with a low share of preference will get greater profit from using long-duration coupons to relax redemption pressure and attract consumers who prefer the rival's brand. Furthermore, a firm is more likely to benefit from a short coupon duration in markets where the proportion of consumers redeeming in expiration-date order is larger.

Other theoretical explanations for the empirically

observed negative relationship between coupon duration and market share may also be offered. One such conjecture is that small share brands may have fewer coupon drops because of a budget constraint, and that they may try to "stretch" their coupons by extending the duration. However, we find that smaller share brands do not have fewer coupon drops (using the cereal and spaghetti sauce FSI coupon data discussed in §2), and when they do (hot dog data) the number of coupon drops has no significant relationship with coupon duration. Therefore, though theoretically possible, this hypothesis does not hold empirically.

Our analysis also shows that the trade-off for increasing coupon duration is not the same as the trade-off for increasing the value of a price deal or a coupon. In the latter situations, the manager trades off the loss from an increased discount to captive consumers, with the gain from attracting new buyers. In increasing coupon duration, however, the coupon value remains the same, but the number of noncoupon buyers decreases. This decrease in the noncoupon buyers does not come about because consumers who were purchasing the brand at full price now purchase it with a coupon, but because consumers who were purchasing the brand at full price now purchase the rival brand with a coupon.

To retain analytic tractability, our model relied on several assumptions. It is important to examine whether major insights stemming from the model remain valid when such assumptions are relaxed.[13]

We assume that a brand's coupon arrival probabilities follow a Bernoulli distribution. However, all derived propositions can be shown to hold in a three-period model where distribution probabilities differ in each period. Thus, the model's main implications are contingent neither on the Bernoulli assumption nor steady-state analysis.

Our analysis also assumes that coupons for different brands are distributed independently so that there is often an overlap between coupons of different brands. Empirical data confirms the existence of coupon overlap. For instance, among FSI coupons for breakfast cereals that we collected, the average duration is 68.6 days. All coupons overlap with at least one other coupon. On average, a coupon overlaps with 96 other coupons. Also, an average coupon overlaps with at least one other coupon for 99.9% of its life. The mean number of days of overlap between any two brands is 40 days. Thus, it is evident that coupon overlapping is common in the marketplace.[14]

It was further presumed that coupon users do not redeem coupons to stockpile a good, which seems to limit the model's applicability to perishable or infrequently purchased products, for which stockpiling cost is high. However, it is easily demonstrated that the present results do not change qualitatively if a proportion of the market engages in stockpiling.[15]

Our model is built for two brands. However, there are many brands in the market from which any one consumer may be purchasing a small set. It can be shown that our results carry over naturally for the three-brand case.

Our research is especially relevant today as an initial, necessary step toward modeling a comprehensive couponing strategy for managers as they reexamine their couponing policy (*Wall Street Journal* 1996, 1997). Our analysis shows the value of experimenting and taking a proactive approach with coupon expiration date. Such a proactive approach may indeed be key to the profitability of coupon promotions that are still immensely popular among U.S. households.[16]

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[14] We thank an anonymous reviewer for suggesting this analysis.

[15] We thank an anonymous reviewer for this suggestion.

Appendix

A. Individual Coupon Redemption Model. We first set up a stylized two-period model for this decision that provides clear intuition about which factors drive the redemption decision. We then show that the same intuition holds in an infinite-horizon model. Without loss of generality, *we assume that coupons arrive at the*

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[13] Demonstrations of the assertions below dealing with relaxation of various model assumptions appear in Technical Appendices D, E, and F, which are available from the authors upon request.

end of a period, and redemption decisions are made at the beginning of a period. We also assume that the preferred brand is A.

Two-Period Model. Assume that the consumer starts Period 1 with an expiring coupon for Brand B and a nonexpiring coupon for Brand A. We drop the individual subscript for simplicity of notation.

Let γ denote the consumer's subjective discounting rate for future coupon value. This includes factors such as the consumer's opportunity cost of money, consumer patience, and the probability of losing or forgetting a coupon. Let s_i denote the consumer's subjective assessment of coupon arrival rate for brand i. Here, s_i is most likely a function of the general long-term couponing environment for the product category, or even across all product categories, and is less likely to be an accurate reflection of individual brands' coupon arrival rates. Prior promotion research shows that most consumers do not have accurate perceptions of individual brands' deal frequencies (Krishna et al. 1991).

The consumer aims to maximize the benefits from coupon redemption for the two periods, Periods 1 and 2. If the consumer redeems in brand-preference order, her(his) net benefits from coupons for the two periods are $NCV^1 = f + \gamma[s_A f + s_B(1 - s_A)(f - d)]$. If the consumer redeems in expiration-date order, her(his) net benefits for the two periods are $NCV^1 = f - d + \gamma f$.

The consumer will redeem in brand-preference order if:

$$f + \gamma[s_A f + s_B(1 - s_A)(f - d)] > f - d + \gamma f.$$

From the inequality, it is clear that both rules may be optimal depending on the relative parameter values. Discounting has a very strong impact on the choice of redemption rule. If the consumer totally discounts the future ($\gamma = 0$), which happens, for example, if the consumer feels (s)he is surely to lose a coupon, the consumer will definitely use brand-preference order. If the consumer does not discount the future at all ($\gamma = 1$), the consumer will definitely use expiration-date order if the consumer thinks that the likelihood of getting a preferred coupon in the next period is small. In the latter situation, by using the expiration-date rule, the consumer utilizes the "option value" of a longer expiration date. She or he will still redeem *all* the coupons that (s)he gets for her(his) preferred brand before they expire, but minimizes the chance of purchasing without a coupon.

Infinite-Horizon Model. Now, we show that the same intuition holds in an infinite horizon model. Let A_i (B_i) denote a coupon for Brand A (B) expiring in i periods. Let NCV^t denote the net current benefits in period t of all future coupon usage. For brevity, we only take the most comprehensive situation where both coupons have long durations. The following are the coupon portfolios/states that a consumer can be in at any time t: (1) no coupons; (2) A_1; (3) A_2; (4) B_1; (5) B_2; (6) A_1, B_1 (B_1 will be discarded by the consumer); (7) A_1, B_2; (8) A_2, B_1 (this is the portfolio requiring a complex decision by the consumer); (9) A_2, B_2; (10) A_1, A_2; and (11) B_1, B_2.

Note the following two facts, which will facilitate our derivations. First, for a rational consumer, any state with more than two coupons is equivalent to a state with two coupons by discarding the "least value" excess coupons, i.e. $A_1, B_2, A_2 = A_1, A_2$; $B_1, B_2, A_2 = A_2, B_2$. This is because all coupons expire in two periods. Second, NCV^t

is bounded. To see this, note that the maximum face value that a coupon can have in any period is f. Therefore, with discounting of the future, the benefit from coupon usage in each period must be less than f. Hence, the total benefits from coupon usage must be less or equal than $\sum_{i=0}^{\infty} \gamma^i f = f/(1 - \gamma)$.

Now, we derive NCV^1 for each of these states.

Let NCV^1 (no coupons) $= x$, $NCV^1(A_1, A_2) = y$, and $NCV^1(B_1, B_2) = z$. Then, we have $NCV^1(A_1) = NCV^1(A_2) = NCV^1(A_1, B_1) = f + x$, $NCV^1(B_1) = NCV^1(B_2) = f - d + x$; and $NCV^1(A_2, B_2) = NCV^1(A_1, B_2) = d + z$. The first two equalities are straightforward. The last one requires some explanation. The only difference in value between either portfolio (A_2, B_2) or (A_1, B_2) and portfolio (B_1, B_2) is that with either of the former two portfolios the consumer will redeem a coupon for Brand A in Period 1, rather than a coupon for Brand B. Thus, (A_2, B_1) and (A_2, B_2) give an additional value of d over a (B_1, B_2) portfolio, which has a value of z.

For state (A_2, B_1), the optimal policy determines whether the consumer redeems in brand-preference order or in expiration-date order.

If the consumer redeems in brand-preference order, we have $NCV^1_{br-pref}(A_2, B_1) = NCV^1(A_2) = f + x$. If the consumer redeems in expiration-date order, we have $NCV^1_{exp-date}(A_2, B_1) = y - d$. Since both $NCV^1_{br-pref}(A_2, B_1)$ and $NCV^1_{exp-date}(A_2, B_1)$ are bounded, the consumer will redeem in brand-preference order if

$$NCV^1_{br-pref}(A_2, B_1) - NCV^1_{exp-date}(A_2, B_1) > 0.$$

To determine this inequality, we need to determine x and y first. We now write out the equations for x, y, and z and solve for three simultaneous equations in three unknowns.

NCV^1 (no coupons) $= x = 0$

$$+ \gamma\{s_A s_B (d + z) + s_A(1 - s_B)(f + x)$$
$$+ s_B(1 - s_A)(f - d + x)$$
$$+ (1 - s_A)(1 - s_B)x\}, \tag{1}$$

$NCV^1(A_1, A_2) = y = f$

$$+ \gamma\{s_A y + (1 - s_A)(1 - s_B)(f + x)$$
$$+ (1 - s_A)s_B(d + z)\}, \tag{2}$$

$NCV^1(B_1, B_2) = z = f - d$

$$+ \gamma\{s_A s_B(d + z) + (1 - s_A)s_B z$$
$$+ s_A(1 - s_B)NCV^1(B_1, A_2)$$
$$+ (1 - s_A)(1 - s_B)(f - d + x)\}. \tag{3}$$

Since $NCV^1(B_1, A_2)$ has a different value depending on whether the consumer is redeeming by expiration-date order or brand-preference order, we need to solve for x, y, and z from Equations (1)–(3) separately for the expiration-date order case (using $NCV^1_{exp-date}(B_1, A_2)$ in Equation (3)) and brand-preference order case (using $NCV^1_{br-pref}(B_1, A_2)$ in Equation (3)). By proper substitution, we have:

Table A1   Long-Term Purchase Probabilities for Different Coupon Redemption Heuristics*

	Purchase Probabilities for Consumers who Redeem in "Order of Brand Preference"		
Brand Preference	Buy A With Coupon	Buy B With Coupon	Buy Preferred Brand Without Coupon
	Environment: Short-Duration Coupon for Brand A, Long Duration Coupon for Brand B		
Brand A	a_A	$(1 - a_A)[p_B^0 + a_B(1 - p_B^0)]$	$(1 - a_A)[1 - p_B^0 - a_B(1 - p_B^0)]$
Brand B	$a_A(1 - a_B)$	a_B	$1 - a_A - a_B + a_A a_B$
	Environment: Long-Duration Coupon for Both Brands		
Brand A	a_A	$(1 - a_A)[p_B^0 + a_B(1 - p_B^0)]$	$(1 - a_A)[1 - p_B^0 - a_B(1 - p_B^0)]$
Brand B	$(1 - a_B)[p_A^0 + a_A(1 - p_A^0)]$	a_B	$(1 - a_B)[1 - p_A^0 - a_A(1 - p_A^0)]$
	Purchase Probabilities for Consumers who Redeem in "Order of Expiration Date"		
	Environment: Short-Duration Coupon for Brand A, Long-Duration Coupon for Brand B		
Brand A	a_A	$(1 - a_A)[p_B^0 + a_B(1 - p_B^0)]$	$(1 - a_A)[1 - p_B^0 - a_B(1 - p_B^0)]$
Brand B	$a_A(1 - p_B^0)$	a_B	$1 - a_A - a_B + a_A p_B^0$
	Environment: Long-Duration Coupon for Both Brands		
Brand A	a_A	$r_B(1)$	$1 - a_A - r_B(1)$
Brand B	$r_A(0)$	a_B	$1 - a_B - r_A(0)$

* a_i is the probability for a coupon user to receive a coupon for Brand i, p_i^0 is the steady-state probability of holding an expiring coupon for Brand i by those who prefer the rival brand, and $r_A(0)$ ($r_B(1)$) is the redemption rate for Brand A (Brand B) from those who prefer Brand B (Brand A).

$$NCV_{br\text{-}pref}^1(A_2, B_1) - NCV_{exp\text{-}date}^1(A_2, B_1)$$

$$= \{[-1 + \gamma^2(-1 + s_A + s_A^2(-1 + s_B)s_B + \gamma(1 + s_B - s_A s_B)]$$

$$[(-1 + \gamma(s_A + 2s_B - 2s_A s_B) + \gamma^2(-1 + s_A)s_B(1 - s_A + s_A s_B))d$$

$$- \gamma(-1 + s_A)(-1 + s_B)(-1 + \gamma s_A s_B)f]\}/$$

$$\{(-1 + \gamma)(1 + \gamma(-1 + s_A)s_B)(-1 + \gamma(s_A + s_B - s_A s_B)$$

$$+ \gamma^2 s_A s_B(-s_B + s_A(-1 + 2s_B)))\}. \tag{4}$$

It is easy to see that either rule (brand-preference order or expiration-date order) may be optimal depending on relative parameter values. For instance, if $s_A = s_B = 0.6$, $d = 0.1$, and $f = 0.6$, we have $NCV_{br\text{-}pref}^1(A_2, B_1) - NCV_{exp\text{-}date}^1(A_2, B_1) = 0.1 > 0$ when $\gamma = 0$, but $NCV_{br\text{-}pref}^1(A_2, B_1) - NCV_{exp\text{-}date}^1(A_2, B_1) = -0.1 < 0$ when $\gamma = 0.82$. In fact, with these parameter values, all consumers with $\gamma < 0.59$ will redeem coupons by brand-preference order and the rest by expiration-date order.

B. Steady-State Purchase Probabilities When Brand B Has a Long-Duration Coupon. The case of short duration for Brand A and long duration for Brand B is symmetric to that of long duration for A and short duration for B in the text and is presented in Table A1. The last case, long coupon duration for both brands, is straight-

forward when the consumer redeem coupons by brand preference. It is, however, more difficult to derive all the relevant probabilities in Table A1 when the consumer redeems by expiration date. We derive them below.

When both brands' coupons have a long duration, coupon users have more leeway in adjusting their redemption sequence. In this case, the probability of a coupon user holding an expiring coupon for a brand will depend on which brand is preferred. For simplicity of notation, let λ be the probability of a coupon user preferring Brand A, and $1 - \lambda$ be the probability of preferring Brand B, where λ assumes a value of either 1 or 0. At any time t, the probability of a coupon user with redemption preference λ having an expiring coupon for Brand A (Brand B), denoted by $p_A^0(\lambda)$ ($p_B^0(\lambda)$), is equal to the probability of the coupon user holding Brand A's (Brand B's) coupon in the previous period $t - 1$, but choosing not to redeem it in period $t - 1$.[17] The probability of the coupon user holding a coupon for Brand A (Brand B) in period $t - 1$ that expires in the following period is simply the arrival rate of Brand A's (Brand B's) coupons, a_A (a_B). Given a coupon for Brand A (Brand B) in period $t - 1$, a coupon user must have chosen not

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[17] $p_i^0(\lambda)$ is similar to p_i^0 in the text except that a user's redemption preference is also taken into account in computing the probability of having an expiring coupon for Brand i.

to use the coupon in that period if she had an expiring coupon for either Brand A or Brand B, or if she chose to redeem Brand B's (Brand A's) coupon because of her brand preference. Therefore, $p_A^0(\lambda)$ and $p_B^0(\lambda)$ are implicitly defined by the following two equations.

$$p_A^0(\lambda) = a_A\{[p_A^0(\lambda) + p_B^0(\lambda) - p_A^0(\lambda)p_B^0(\lambda)]$$
$$+ [1 - p_A^0(\lambda)][1 - p_B^0(\lambda)]a_B(1 - \lambda)\}, \quad \text{(B.1)}$$

$$p_B^0(\lambda) = a_B\{[p_A^0(\lambda) + p_B^0(\lambda) - p_A^0(\lambda)p_B^0(\lambda)]$$
$$+ [1 - p_A^0(\lambda)][1 - p_B^0(\lambda)]a_A\lambda\}. \quad \text{(B.2)}$$

A coupon user with redemption preference λ will redeem a coupon for Brand A (Brand B) on a purchase occasion in four exclusive events: if (s)he has a coupon for Brand A (Brand B), but has no coupon for Brand B (Brand A); if (s)he has both brands' coupons expiring in the current period, but chooses to buy Brand A (Brand B) because of brand preference; if (s)he has a coupon for Brand A (Brand B) expiring in the current period, a coupon for Brand B (Brand A) expiring in the next period, and no Brand B (Brand A) coupon expiring in the current period; and, finally, if (s)he has coupons for both brands expiring in the next period, has no Brand A or Brand B coupon expiring in the current period, and purchases Brand A (Brand B) because of brand preference.

Summing up the probabilities for these four events, the redemption rates for the two brands from a coupon user with brand preference λ, denoted by $r_A(\lambda)$ and $r_B(\lambda)$ respectively, are given by

$$r_A(\lambda) = \{1 - (1 - a_A)[1 - p_A^0(\lambda))]\}\{(1 - a_B)[1 - p_B^0(\lambda)]\}$$
$$+ p_A^0(\lambda)p_B^0(\lambda)\lambda + p_A^0(\lambda)a_B[1 - p_B^0(\lambda)]$$
$$+ a_Aa_B[1 - p_A^0(\lambda)][1 - p_B^0(\lambda)]\lambda,$$

$$r_B(\lambda) = \{1 - (1 - a_B)[1 - p_B^0(\lambda)]\}\{(1 - a_A)[1 - p_A^0(\lambda)]\}$$
$$+ p_A^0(\lambda)p_B^0(\lambda)(1 - \lambda) + p_B^0(\lambda)a_A[1 - p_A^0(\lambda)]$$
$$+ a_Aa_B[1 - p_A^0(\lambda)][1 - p_B^0(\lambda)](1 - \lambda).$$

It can be easily shown that the redemption probability for Brand A's (Brand B's) coupons from a coupon user who prefers the brand is reduced to $r_A(1) = a_A$ ($r_B(0) = a_B$). Similarly, one can show that

$$r_A(0) = \{1 - (1 - a_A)[1 - p_A^0(0))]\}\{(1 - a_B)[1 - p_B^0(0)]\}$$
$$+ p_A^0(0)a_B[1 - p_B^0(0)],$$

$$r_B(1) = \{1 - (1 - a_B)[1 - p_B^0(1)]\}\{(1 - a_A)[1 - p_A^0(1)]\}$$
$$+ p_B^0(1)a_A[1 - p_A^0(1)].$$

When not redeeming any coupon, the individual purchases the preferred brand without any coupon as shown in Table A1.

C. Steady-State Payoffs and Competitive Equilibrium. Let Π_A^{jk} be Firm A's payoffs when Firm A chooses coupon duration j and Firm B chooses k. On the basis of purchasing incidence in Table 3 in the text and Table A1, we have

$$\Pi_A^{ss} = \alpha\{\theta[a(m - f) + (1 - 2a + a^2)m] + (1 - \theta)a(1 - a)(m - f)\}$$
$$+ (1 - \alpha)\{\theta[a(m - f) + (1 - 2a + a^2)m]$$
$$+ (1 - \theta)a(1 - a)(m - f)\},$$

$$\Pi_A^{ls} = \alpha\{\theta[a(m - f) + (1 - 2a + a^2)m]$$
$$+ (1 - \theta)(1 - a)(p_A^0 + a(1 - p_A^0))(m - f)\}$$
$$+ (1 - \alpha)\{\theta[a(m - f) + (1 - a - a(1 - p_A^0))m]$$
$$+ (1 - \theta)(1 - a)[(1 - a)p_A^0 + a](m - f)\},$$

$$\Pi_A^{sl} = \alpha\{\theta[a(m - f) + (1 - a)^2(1 - p_B^0)m]$$
$$+ (1 - \theta)a(1 - a)(m - f)\} + (1 - \alpha)\{\theta[a(m - f)$$
$$+ (1 - a)^2(1 - p_B^0)m] + (1 - \theta)a(1 - p_B^0)(m - f)\},$$

$$\Pi_A^{ll} = \alpha\{\theta[a(m - f) + (1 - a)^2(1 - p_B^0)m]$$
$$+ (1 - \theta)(1 - a)(p_A^0 + a(1 - p_A^0))(m - f)\}$$
$$+ (1 - \alpha)\{\theta[a(m - f) + (1 - a - r_B(1))m]$$
$$+ (1 - \theta)r_A(0)(m - f)\}.$$

The profit functions for Firm B are symmetric.

The first equation above, for instance, is derived in the following manner. Purchase probabilities for the (s, s) strategy are given in Table 3 in the text. Firm A's profits come from four segments of coupon users: those who belong to α and prefer either Brand A or Brand B, and those who belong to $(1 - \alpha)$ and prefer either Brand A or Brand B. The profits that Firm A gets from those who belong to α and prefer Brand A are given by $\alpha\theta$, the size of the group, multiplied by $a(m - f) + (1 - 2a + a^2)m$, the profit margins weighted by the respective purchase probabilities. We can similarly go through the other three segments.

Note that symmetry implies $p_A^0 = p_B^0$ and $r_A(0) = r_B(1)$. Let us define

$$\delta_1 = r_B(1) - \frac{1 - a}{a} p_B^0,$$

$$\delta_2 = r_A(0) - \frac{1 - a}{a} p_B^0 - \alpha\left(r_A(0) - \frac{1 - a}{a} p_B^0(1 + a - a^2)\right),$$

$$\theta_1 = \frac{m - f}{(m - f) + (1 - \alpha)m},$$

$$\theta_2 = \frac{\delta_2(m - f)}{\delta_2(m - f) + \delta_1(1 - \alpha)m},$$

where $0 < \delta_1 < 1$, $0 < \delta_2 < 1$, $\delta_2 > \delta_1$, and $\theta_2 > \theta_1$. Some algebra will show $\Pi_A^{ss} - \Pi_A^{ls} > 0$ if $\theta > \theta_1$ and $\Pi_A^{sl} - \Pi_A^{ll} > 0$ if $\theta > \theta_2$. Using symmetry (substituting $1 - \theta$ for θ), we have $\Pi_B^{ss} - \Pi_B^{sl} > 0$ if $\theta < 1 - \theta_1$ and $\Pi_B^{ls} - \Pi_B^{ll} > 0$ if $\theta < 1 - \theta_2$. Thus, we can list all possible equilibria for the stage game in Table A2. Figure A1 summarizes the conditions under which an specific equilibrium occurs. We now show the details of their derivations.

Table A2   Nash Stage Equilibrium Conditions

(s, s)	$\begin{cases} \theta > \theta_1 \\ \theta < (1 - \theta_1) \end{cases}$
(l, s)	$\begin{cases} \theta < \theta_1 \\ \theta < (1 - \theta_2) \end{cases}$
(s, l)	$\begin{cases} \theta > \theta_2 \\ \theta > (1 - \theta_1) \end{cases}$
(l, l)	$\begin{cases} \theta < \theta_2 \\ \theta > (1 - \theta_2) \end{cases}$

Because $\theta_2 > \theta_1$, we necessarily have $1 - \theta_1 > 1 - \theta_2$. If $\theta_1 > \frac{1}{2}$, i.e., if $\alpha > f/m$, we must have $\theta_1 > (1 - \theta_1)$. To determine the relative magnitude between θ_1 and $(1 - \theta_2)$, we define $G(\alpha) = \theta_1 + \theta_2$. Since $\partial \theta_1 / \partial \alpha > 0$ and $\partial \theta_2 / \partial \alpha > 0$, we must have $G'(\alpha) > 0$. Furthermore, we have $G(0) = [2(m - f)/((m - f) + m)] < 1$ and $G(1) = 2$. These conditions imply that there exists a unique α_1^*, such that $G(\alpha_1^*) = 1$. Thus, for $\alpha > \alpha_1^*$, we necessarily have $\theta_1 > (1 - \theta_2)$. The relative magnitude between θ_2 and $1 - \theta_2$ can also be determined similarly. Note that if $\alpha = 1$ we have $\theta_2 = 1$, and if $\alpha = 0$ (implying $\delta_1 = \delta_2$) we have $\theta_2 < \frac{1}{2}$. There must exist a unique α_2^* such that $\theta_2(\alpha_2^*) = \frac{1}{2}$ and $\alpha_2^* > 0$. Thus, for any $\alpha > \alpha_2^*$, we have $\theta_2 > 1 - \theta_2$.

Figure A1   Equilibrium Coupon Duration

(a): $\alpha > \frac{f}{m}$

(b): $\alpha_1^* < \alpha < \frac{f}{m}$

(c): $\alpha_2^* < \alpha < \alpha_1^*$

(d): $\alpha < \alpha_2^*$
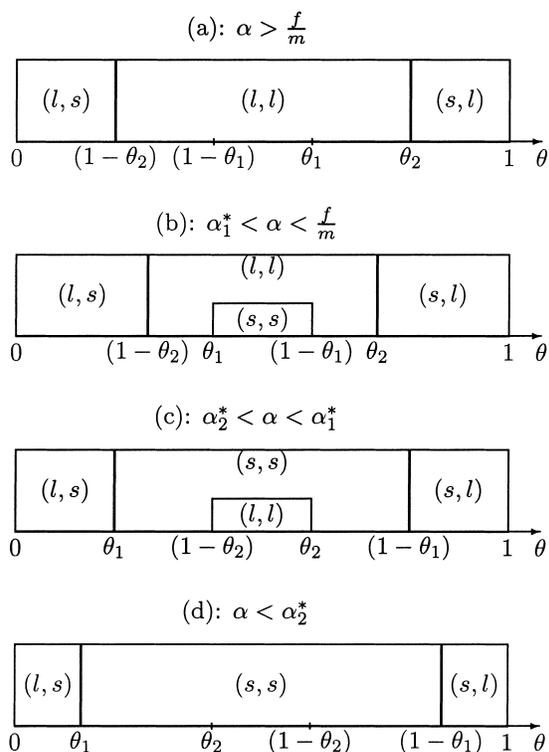
Finally, we must have $\alpha_1^* < f/m$. The reason is that at $\alpha = f/m$, we have $\theta_1 = \frac{1}{2}$, which implies $G(f/m) > 1$ since $\theta_2 > \theta_1$. Furthermore, we must have $\alpha_2^* < \alpha_1^*$. The reason is that at α_2^*, we have $\theta_2 = \frac{1}{2}$ which implies $G(\alpha_2^*) < 1$ since $\theta_1 < \theta_2$. In sum, we have $0 < \alpha_2^* < \alpha_1^* < f/m < 1$.

The above analysis allows us to fully characterize the equilibria for any given α. For instance, given $1 > \alpha > f/m$, we must have $(1 - \theta_2) < (1 - \theta_1) < \theta_1 < \theta_2$. By using the equilibrium conditions, we can determine the equilibrium of the stage game for any given θ, which is illustrated in Figure A1. We can go through the same procedure for other three ranges of α. The results are illustrated in Figures A1 (b) to (d). *Q.E.D.*

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